Topic Study Group 8

Teaching and Learning of Geometry at Primary Level

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1. Aims of the TSG

This group provided a forum for discussion of the learning and teaching of geometry, with a focus on the elementary grades, K-6 (or preK-8). We had short presentations on, and discussions of, important new trends and developments in research or practice, in geometry teaching and learning, and expositions of outstanding recent contributions to it.

The focus of the group was on theoretical, empirical or developmental issues related to the themes below. The issues raised were considered from the historical, epistemological and ontological, cognitive and semiotic, and educational points of view. The following subthemes are proposed:

- Studies of the use of new/alternate geometry curricula or curriculum components (including topological ideas, ethnomathematical approaches, etc.);
- The use of tools/resources such as physical manipulatives (e.g., pattern blocks, cubes, paper folding, mirrors) and digital technologies;
- Problem solving in geometric contexts;
- Task design for the teaching and learning of elementary geometry;
- Explanation, argumentation, and proof in elementary geometry education;
- Spatial and geometric reasoning about two- and three-dimensional shapes;
- Psychological roots of spatial, visual and geometrical thinking;
- The role of geometrical transformations in learning and teaching geometry;
- Teacher knowledge and preparation in geometry education.

1.1. Submissions

We received 16 submissions from 12 countries (Canada 1, China 2, Japan 2, Great Britain 1, France 1, Austria 1, Denmark 1, Switzerland 1, Hungary 1, Turkey 2,

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Australia 1, New Zealand 2), thus reaching our goal of diverse cultural representation. Of those 16 submissions, 8 were accepted as long orals and 8 as short orals.

1.2. Sessions

In our first 90-minute session, the TSG co-Chair, Michael Battista, introduced the rest of the Team and described the format of the sessions. The second 90-minute session was introduced by co-Chair Nathalie Sinclair. The third 120-minute session was introduced by TSG-8 members Eszter Kónya and Haiyue Jin. Generally, all three sessions led with two or three long oral presentation and discussion and two short oral presentations with a short collective discussion. Throughout the three days, we encouraged participants to engage in discussion around each presentation, as well as across the presented papers.

1.3. Paper Topics

Of the 16 accepted papers, only 13 papers were able to be presented during the online conference. There were seven long oral presentations and six short oral presentations. A list of these papers and authors are included in order of presentation and are organized in Tab. 1.

Tab. 1. List of papers presented

Paper and author(s)	
[1]	Mathematical knots as a teaching material to improve pupils spatial abilities in elementary school. ***Tomoko Yanagimoto****, Akiyo Higasio, Madoka Koyama, Hisashi KinoShita, and Moe Miyazaki* (Japan).
[2]	The transition from informal to formal area measurement. ***Eszter Herendiné-Kónya*** (Hungary).
[3]	Tilings and symmetry using the labosaique. *Paolo Bellingeri,* ***Emmanuelle Feaux De Lacroix****, Eric Reyssat, and Andre Sesboue* (France).
[4]	The knowledge to be taught: a novice mathematics teacher plans to teach quadrilaterals in 5th grade. ***Nazlı Akar*** *and Mine Işıksal Bostan* (Turkey).
[5]	Supporting the development of young children's spatial reasoning: insights from the math for young children (m4yc) project. ***Catherine Diane Bruce,*** *Zachary Hawes, and Tara C Flynn* (Canada).
[6]	The basic routines of spatial thinking and acting. ***Marion Zoggeler*** (Austria).
[7]	Developing spatial abilities and geometrical knowledge with use of a virtual city. *Jean-Luc Dorier and* ***Sylvia Coutat*** (Switzerland).
[8]	Understanding path descriptions in a Manhattan-like map — a comparison of German 2nd and 3rd graders. ***Elisabeth Mantel*** (Germany).
[9]	Impact of teacher professional learning on students geometric reasoning relating to prisms. ***Ann Patricia Downton*** (Australia).
[10]	Unpacking language in geometry lesson on shapes in a New Zealand multilingual primary class. ***Shweta Sharma*** (New Zealand).
[11]	Spatial visualisation reasoning about 2d representations of 3d geometrical shapes: the case of G4–6. ***Taro Fujita****, Yutaka Kondo, Hiroyuki Kumakura, Susumu Kunimune, and Keith Jones* (UK).
[12]	Exploring second graders performances on reading comprehension of mathematics picture book with words and no-word. ***Yan-Hong Chen*** (China).
[13]	Implementing the project-based approach in teaching the area of circle: An explorative study. ***Jinyu Yu*** (China).

2. Conference Themes

The highest proportion of papers focused on the subtheme of spatial reasoning, with two long orals and two short orals presented on the second day. While two of these papers related specifically to geometry, two others were broader in terms of their mathematics curriculum connection. It seems clear that spatial reasoning has become a significant area of attention, and that it can offer an interesting opportunity for connecting the arithmetic/algebra and geometry components of curriculum. The researchers all emphasized the strong correlations between spatial reasoning and success in school mathematics, while also pointing to the relative paucity of focus on spatial reasoning activities in most primary school mathematics classroom. The tasks studied across the four research presentations covered a variety of contexts (using physical materials as well as digital technology; occurring in both indoor and outdoor situations). The third day's presentations also included presentations on spatial reasoning skills. In addition, the role of everyday language in concept formation was discussed, with a special focus on multilingual classrooms. 2D-3D visualization, interpreting representations, and switching between representations were also discussed. The presentations also touched on related areas of teacher training beyond the teaching of geometry in primary schools.

We list below the main TSG-8 Meeting Themes/Research Questions

- How can instruction support the learning of geometric measurement and formulas relationally (conceptually) in a way that is integrated with procedural fluency?
- What ways of visualizing and representing (including verbally describing) enable students to understand the structure of 3D shapes?
- How is spatial structuring — in say, tiling and using isometries — related to knowledge of geometric properties?
- How does the depth of teacher knowledge of geometric content affect how they teach that content?
- How can mathematics education researchers productively investigate ideas in cognitive science spatial ability research in ways that reveal students' actual sense making in mathematics contexts? That is, how can mathematics education researchers (perhaps in collaboration with cognitive scientists) use qualitative research to elaborate and deepen the knowledge produced in spatial reasoning quantitative research?

3. Areas for Future Research

With respect to spatial reasoning, there has emerged a variety of ways of describing, characterizing and identifying instances of it. The conference offered an excellent opportunity for researchers to find some convergence and overlaps, and continued work in this direction would facilitate future research collaboration. While spatial reasoning is recognized as an important aspect of mathematical thinking, more research on how the spatial reasoning activities are used by teachers in ways that support specific conceptual development is warranted. Finally, greater attention to the theoretical framing of spatial reasoning would help researchers better understand how engaging visual and kinaesthetic ways of thinking can support learning — in particular, theories of embodied cognition could offer much to future research in this area.

As preschool and primary school children are already part of the alpha generation, it would be important to extend research on how digital manipulations can contribute to a better understanding of geometric concepts. What differences in cognitive development are caused when physical manipulative activities are replaced by digital applications in early childhood?

4. TSG-8 Future Research Ideas

Research investigating interrelationships between spatial and geometric reasoning is essential — unfortunately, it has been neglected in recent mathematics education research. Future research should include:

- detailed research that investigates, from a cognitive perspective, exactly how spatial reasoning supports geometric reasoning, and vice versa.
- an understanding of how visualization and geometric property knowledge become integrated to form abstract geometric reasoning that is grounded in students' real-world experiences, and how instruction can support this integration.
- investigating the role that mental models play in mathematical reasoning, with spatial reasoning playing a critical role in constructing and operating on mental models.
- going beyond general theories of relationships between spatial and geometric reasoning to detailed elaborations of spatial-geometric interrelationships for specific topics in geometry (which may lead back to general theories).
- investigating, with student interviews and teaching experiments, the nature of, and causes for, the correlation between spatial and mathematical reasoning found by cognitive psychologists, moving towards understanding underlying reasons for how spatial reasoning is related to, and supports, mathematics reasoning for various topics.
- investigating area measurement through decomposition and composition, that is, to consider the possibility of finding the measure of the area of a polygon from its reconfigurations. Also, the existence of other variables that favour the decomposition and composition operation, such as the use of the grid mesh, can be incorporated in other studies.
- carrying out more research about the learning of 3D geometry, because spatial geometry, deserves a different reflection and analysis, since drawing representations of three-dimensional figures, on the board or in paper, does not allow the student to fully observe the characteristics and properties.